מד"ר 1 / תרגיל בית 1#

אריאל סטולרמן

1. $y' = y$:

$(a)\ \frac{dy}{dx} = 1 \cdot y \Rightarrow \int \left(\frac{1}{y}\right) dy = \int dx \Rightarrow ln|y| = x + c' \Rightarrow y = e^{x+c'} \underset{e^{c'}:=c}{\Rightarrow} y = c \cdot e^x$

$(b)\ y(-2) = 4 \Rightarrow 4 = c \cdot e^{-2} \Rightarrow c = 4e^2 \Rightarrow y = 4e^{x+2}$

$(c)\ y' = y:\ y' = (c \cdot e^x)' = c \cdot e^x = y$

2. $x^2y' + y = 0$:

$(a)\ x^2y' + y = 0 \Rightarrow y' = -\frac{y}{x^2} \Rightarrow \frac{dy}{dx} = -\frac{y}{x^2} \Rightarrow \frac{1}{y}dy = -\frac{1}{x^2}dx \Rightarrow \int \frac{1}{y}dy = -\int \frac{1}{x^2}dx \Rightarrow ln|y| = -\left(-\frac{1}{x}\right) + c' = \frac{1}{x} + c' \Rightarrow$

$y = e^{\frac{1}{x}+c'} \underset{c:=e^{c'}}{\Rightarrow} y = c \cdot e^{\frac{1}{x}}$

$(b)\ y(-2) = 4 \Rightarrow 4 = c \cdot e^{\frac{1}{-2}} \Rightarrow c = 4\sqrt{e} \Rightarrow y = 4 \cdot e^{\frac{1}{x}+\frac{1}{2}}$

$(c)\ y' = -\frac{y}{x^2}:\ y' = \left(c \cdot e^{\frac{1}{x}}\right)' = c \cdot e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) = -\frac{c \cdot e^{\frac{1}{x}}}{x^2} = -\frac{y}{x^2}$

3. $(y+1)xy' - (x-1)y = 0$:

$(a)\ y' = \frac{(x-1)y}{(y+1)x} \Rightarrow \frac{dy}{dx} = \frac{(x-1)y}{(y+1)x} \Rightarrow \int \left(1 + \frac{1}{y}\right) dy = \int \left(1 - \frac{1}{x}\right) dx \Rightarrow y + ln|y| = x - ln|x| + c' \Rightarrow ln|y| + ln|x| = x - y + c'$

$\Rightarrow ln|xy| = x - y + c' \Rightarrow xy = e^{x-y+c'} \underset{c:=e^{c'}}{\Rightarrow} xy = c \cdot e^{x-y}$

$(b)\ y(-2) = 4 \Rightarrow -2 \cdot 4 = c \cdot e^{-2-4} \Rightarrow c = -8e^6 \Rightarrow xy = -8e^{x-y+6}$

$(c)\ y' = \frac{(x-1)y}{(y+1)x};\ xy = ce^{x-y} \Rightarrow y = \frac{ce^{x-y}}{x} \Rightarrow y' = \frac{x \cdot ce^{x-y} \cdot (x-y)' - ce^{y-x}}{x^2} = \frac{xce^{y-x}(1-y') - ce^{y-x}}{x^2} =$

$= \frac{ce^{x-y}(x - xy' - 1)}{x^2} = \frac{y(x - xy' - 1)}{x} \Rightarrow xy' = xy - xyy' - y \Rightarrow y'(x + xy) = xy - y \Rightarrow y' = \frac{y(x-1)}{x(y+1)}$

4. $(y+x)y' + y - x = 0$:

$(a)\ (y+x) \cdot \frac{dy}{dx} + (y-x) = 0 \Rightarrow (\underset{M}{y-x})dx + (\underset{N}{y+x})dy = 0;\quad M_y = 1, N_x = 1 \Rightarrow the\ equation\ is\ exact \Rightarrow$

$u(x,y) = \int Mdx + \varphi(y) = \int (y-x)dx + \varphi(y) = xy - \frac{1}{2}x^2 + \varphi(y);$

$u_y = x + \varphi'(y) = N = y + x \Rightarrow \varphi'(y) = y \Rightarrow \varphi(y) = \frac{1}{2}y^2 \Rightarrow solution:\ u = xy - \frac{1}{2}x^2 + \frac{1}{2}y^2 = c' \Rightarrow 2xy - x^2 + y^2 = c$

$(b)\ 2 \cdot (-2) \cdot 4 — 22 + 4^2 = c \Rightarrow c = -4$

$(c)\ F(x,y) = 2xy - x^2 + y^2;\ y' = -\frac{F_x}{F_y} = -\frac{2y - 2x}{2x + 2y} = -\frac{y-x}{x+y} \Rightarrow (y+x)y' = -(y-x) \Rightarrow (y+x)y' + y - x = 0$

5. $xy' - y = \sqrt{x^2+y^2}$:

$(a)\ xy' - y = \sqrt{x^2+y^2} \Rightarrow y' = \sqrt{1 + \left(\frac{y}{x}\right)^2} + \frac{y}{x};\quad u = \frac{y}{x} \Rightarrow y = xu \Rightarrow y' = u + xu' \Rightarrow u + xu' = \sqrt{1+u^2} + u \Rightarrow$

$x \cdot \frac{du}{dx} = \sqrt{1+u^2} \Rightarrow \frac{1}{\sqrt{1+u^2}}du = \frac{1}{x}dx \Rightarrow \int \frac{1}{\sqrt{1+u^2}}du = \int \frac{1}{x}dx \Rightarrow arcsinh(u) = ln\left(u + \sqrt{u^2+1}\right) = ln|x| + \underset{c'=ln\ c}{ln\ c} = ln\ |xc|$

$\Rightarrow u + \sqrt{u^2+1} = xc;\quad y = ux \Rightarrow cy = u^2 + u\sqrt{u^2+1} = \frac{y^2}{x^2} + \frac{y}{x}\sqrt{\frac{y^2}{x^2}+1} \Rightarrow x^2cy = y^2 + xy\sqrt{y^2/x^2+1} \Rightarrow$

$cx^2 - y = x\sqrt{y^2/x^2+1} \Rightarrow c^2x^4 - 2cx^2y + y^2 = x^2\left(\frac{y^2}{x^2}+1\right) = x^2 + y^2 \Rightarrow c^2x^4 - 2cx^2y = x^2 \Rightarrow c^2x^2 - 2cy = 1 \Rightarrow$

$y = \frac{c^2x^2 - 1}{2c}$

$(b)\ y(-2) = 4 \Rightarrow 4 = \frac{4c^2 - 1}{2c} \Rightarrow 4c^2 - 8c - 1 = 0 \Rightarrow c = 1 \pm \sqrt{5/4}$

$(c)\ y' = \frac{\sqrt{x^2+y^2} + y}{x};\quad \left(\frac{c^2x^2 - 1}{2c}\right)' = \frac{2c^2x \cdot 2c - 0}{4c^2} = \frac{2c^2x}{2c} = cx =: (*)$

$$\frac{\sqrt{x^2+y^2}+y}{x}=\frac{1}{x}\left(\sqrt{x^2+\left(\frac{c^2x^2-1}{2c}\right)^2}+\frac{c^2x^2-1}{2c}\right)=\frac{1}{2xc}\left(\sqrt{4c^2x^2+c^4x^4-2c^2x^2+1}+c^2x^2-1\right)=$$
$$\frac{1}{2xc}\left(\sqrt{(c^2x^2)^2+2c^2x^2+1}+c^2x^2-1\right)=\frac{1}{2xc}\left(\sqrt{(c^2x^2+1)^2}+c^2x^2-1\right)=\frac{1}{2xc}(c^2x^2+1+c^2x^2-1)=cx=(*)$$

6. $y^2+x^2y'=xyy'$:

$$y^2+x^2y'=xyy'\Rightarrow y'=\frac{y^2}{xy-x^2}\Rightarrow y'=\frac{(y/x)^2}{y/x-1};\quad u=\frac{y}{x}\Rightarrow y=xu\Rightarrow y'=u+xu'$$
$$y'=u+xu'=\frac{u^2}{u-1}\Rightarrow x\cdot\frac{du}{dx}=\frac{u^2-u^2+u}{u-1}=\frac{u}{u-1}\Rightarrow\left(1-\frac{1}{u}\right)du=\frac{1}{x}dx\Rightarrow\int 1-\frac{1}{u}du=\int\frac{1}{x}dx\Rightarrow u-ln|u|=ln|x|+c$$
$$\Rightarrow\frac{y}{x}-ln\frac{y}{x}=ln\,x+c\Rightarrow\frac{y}{x}=ln\,x+ln\frac{y}{x}+c=ln\,y+c\Rightarrow ln\,y=\frac{y}{x}-c\Rightarrow y=c'\cdot e^{\frac{y}{x}}$$
$(b)\ y(-2)=4\Rightarrow 4=c'\cdot e^{-2}\Rightarrow c'=4e^2\Rightarrow y=4e^{\frac{y}{x}+2}$

$(c)\ -$

7. $(x^2+y^2)y'=2xy$:

$$(a)\ (x^2+y^2)y'=2xy\Rightarrow y'\left(1+\left(\frac{y}{x}\right)^2\right)=2\cdot\frac{y}{x};\ \ u=\frac{y}{x}\Rightarrow y'(1+u^2)=2u\Rightarrow y'=\frac{2u}{1+u^2}\Rightarrow u+xu'=\frac{2u}{1+u^2}\Rightarrow$$
$$x\cdot\frac{du}{dx}=\frac{2u-u-u^3}{1+u^2}=\frac{u-u^3}{1+u^2}\Rightarrow\frac{1+u^2}{u-u^3}du=\frac{1}{x}dx\Rightarrow\int\frac{1}{u-u^3}du+\int\frac{u}{1-u^2}du=\int\frac{1}{x}dx\Rightarrow-ln(u+1)+ln\,u-ln(u-1)$$
$$=ln\,x+c'\Rightarrow ln\frac{u}{u^2-1}=ln\,xc''\Rightarrow xc''=\frac{u}{u^2-1}=\frac{y/x}{(y/x)^2-1}\Rightarrow\frac{y^2}{x}c''-xc''=\frac{y}{x}\Rightarrow y^2c''-x^2c''=y\Rightarrow y^2-x^2=cy$$
$(b)\ 16-4=4c\Rightarrow c=3\Rightarrow y^2-x^2=3y$
$$(c)\ y'=\frac{2xy}{x^2+y^2};\quad y^2-x^2=cy\Rightarrow y=\frac{y^2-x^2}{c}=\frac{1}{c}y^2-\frac{1}{c}x^2\Rightarrow y'=\frac{1}{c}(y^2)'-\frac{1}{c}(x^2)'=\frac{1}{c}\cdot 2yy'-\frac{1}{c}2x\Rightarrow$$
$$y'\left(1-\frac{2y}{c}\right)=-\frac{2x}{c}\Rightarrow y'=\frac{-2x}{c-2y}=\frac{2xy}{2y^2-cy}=\frac{2xy}{2y^2-y^2+x^2}=\frac{2xy}{x^2+y^2}$$

8. $y'-\frac{2y}{x+1}=(x+1)^3$:

$$(a)\ y'+\underbrace{\left(-\frac{2}{x+1}\right)}_{p}y=\underbrace{(x+1)^3}_{q}\Rightarrow linear\ equation\ in\ 1^{st}\ order\Rightarrow z(x)=ye^{\int p(x)dx}=ye^{\int-\frac{2}{x+1}dx}=ye^{-2\cdot ln(x+1)}=ye^{ln\frac{1}{(x+1)^2}}$$
$$\Rightarrow z(x)=\frac{y}{(x+1)^2}\Rightarrow z'(x)=y'\cdot\frac{1}{(x+1)^2}+y\left(-\frac{2}{(x+1)^3}\right)=\frac{1}{(x+1)^2}\cdot\underbrace{\left(y'-\frac{2}{x+1}\right)}_{=q=(x+1)^3}=\frac{(x+1)^3}{(x+1)^2}=x+1\Rightarrow$$
$$z(x)=\int x+1dx=\frac{x^2}{2}+x+c';\quad y\frac{1}{(x+1)^2}=\frac{x^2+2x}{2}+c'=\frac{(x+1)^2}{2}-\frac{1}{2}+c'=\frac{(x+1)^2}{2}+c\Rightarrow y=\frac{(x+1)^4}{2}+c(x+1)^2$$
$$(b)\ 4=\frac{1}{2}+c\Rightarrow c=3.5\Rightarrow y=\frac{(x+1)^4}{2}+3.5(x+1)^2$$
$$(c)\ y'=(x+1)^3+\frac{2y}{x+1}=(x+1)^3+\frac{2}{x+1}\left(\frac{(x+1)^4}{2}+c(x+1)^2\right)=(x+1)^3+(x+1)^3+2c(x+1)=$$
$$2(x+1)^3+2c(x+1);\quad y=\frac{(x+1)^4}{2}+c(x+1)^2\Rightarrow y'=2(x+1)^3+2c(x+1)$$

9. $y+y'=e^{-x}$:

$$(a)\ y'+\underbrace{1}_{p}\cdot y=\underbrace{e^{-x}}_{q}\Rightarrow\ z(x)=ye^{\int dx}=ye^x\Rightarrow z'(x)=y'e^x+ye^x=e^x\underbrace{(y'+y)}_{=q=e^{-x}}=1\Rightarrow z(x)=x+c\Rightarrow$$
$$ye^x=x+c\Rightarrow y=\frac{x+c}{e^x}$$
$$(b)\ 4=(-2+c)e^2\Rightarrow c=\frac{4}{e^2}+2\ ...$$
$$(c)\ y=\frac{x+c}{e^x}\Rightarrow y'=\frac{e^x-(x+c)e^x}{e^{2x}}=\frac{1-x-c}{e^x};\quad y+y'=e^{-x}\Rightarrow y'=\frac{1}{e^x}-y=\frac{1}{e^x}-\frac{x+c}{e^x}=\frac{1-x-c}{e^x}$$

10. $(1+x)y'-[2y+(x+1)^4]=0$:

$(1+x)y'-2y-(x+1)^4=0\Rightarrow y'-\frac{2}{x+1}y=(x+1)^3\Rightarrow$ פתרון תרגיל 8

11. $x\,ln\,x\cdot y'-y=x^3(3\,ln\,x-1)$:

$$y'-\underbrace{\frac{1}{(xlnx)}}_{p}y=\frac{3x^3lnx-x^3}{xlnx}=\underbrace{3x^2-\frac{x^2}{lnx}}_{q};\quad z(x)=ye^{\int p(x)dx}=ye^{\int-\frac{1}{xlnx}dx}$$

$$\int -\frac{1}{xlnx}dx = -\int \frac{1}{lnx}\cdot\frac{1}{x}dx = -\int \frac{1}{lnx}\cdot(lnx)'dx = \begin{bmatrix} f\equiv ln \\ g\equiv ln \end{bmatrix} = -\int f'\big(g(x)\big)\cdot g'(x)dx = -\ln\ln x \Rightarrow$$

$$z(x) = ye^{-\ln\ln x} = \frac{1}{\ln x}y \Rightarrow z'(x) = \frac{1}{\ln x}y' - \frac{1}{x(lnx)^2}y = \frac{1}{lnx}\Big(\underbrace{y' - \frac{1}{xlnx}y}_{=q=3x^2-\frac{x^2}{lnx}}\Big) = \frac{1}{lnx}\left(3x^2 - \frac{x^2}{lnx}\right) = \frac{3x^2}{lnx} - \frac{x^2}{(lnx)^2} \Rightarrow$$

$$z(x) = \int \frac{3x^2}{lnx} - \frac{x^2}{(lnx)^2}dx = \int \frac{3x^2 lnx - x^2}{(lnx)^2}dx = \int \frac{3x^2\cdot lnx - x^3\cdot\frac{1}{x}}{(lnx)^2}dx = \begin{bmatrix} f\equiv x^3 \\ g\equiv lnx \end{bmatrix} = \int \left(\frac{f}{g}\right)' dx = \frac{f}{g} + c = \frac{x^3}{lnx} + c \Rightarrow$$

$$\frac{x^3}{\ln x} + c = \frac{1}{lnx}y \Rightarrow y = x^3 + clnx$$

(b) ...

(c)

12. $s'\cos t + s\cdot\sin t = 1$:

$$(a)\ s' + s\cdot\underbrace{\frac{sint}{cost}}_{p} = \underbrace{\frac{1}{cost}}_{q};\ \ z(t) = s(t)\cdot e^{\int\frac{sint}{cost}dt}$$

$$\int \frac{sint}{cost}dt = -\int \frac{1}{cost}\cdot(-sint)dt = \begin{bmatrix} f'(t)\equiv\frac{1}{t}\Rightarrow f\equiv \ln t \\ g'(t) = -sint \Rightarrow g(t) = cost \end{bmatrix} = -\int f'\big(g(t)\big)\cdot g'(t)dt = -f\cdot g(t) + c = -\ln(cost) + c$$

$$\Rightarrow z = se^{-\ln cost} = se^{\ln\frac{1}{cost}} = \frac{1}{cost}s \Rightarrow z' = \frac{sint}{(cost)^2}s + \frac{1}{cost}s' = \frac{1}{cost}\Big(\underbrace{s' + \frac{sint}{cost}s}_{=q=\frac{1}{cost}}\Big) = \frac{1}{(cost)^2} \Rightarrow$$

$$z = \int \frac{1}{(cost)^2}dt = tant + c \Rightarrow \frac{1}{cost}s = \frac{sint}{cost} + c \Rightarrow s = sint + c\cdot cost$$

(b) ...

(c) ...

13. $y' + 2y = y^2e^x$:

$$(a)\ y^{-2}y' + \underbrace{2}_{p}y^{-1} = \underbrace{e^x}_{q};\ \ z = y^{-1} \Rightarrow z' = -y^{-2}y' \Rightarrow -z' = y^{-2}y'$$

$$-z' + 2z = e^x \Rightarrow z' + \underbrace{(-2)}_{p}z = \underbrace{-e^x}_{q};\ \ u = ze^{\int -2dx} = ze^{-2x} \Rightarrow u' = z'e^{-2x} - 2ze^{-2x} = e^{-2x}\underbrace{(z' - 2z)}_{=q=-e^x} = e^{-2x}(-e^x) = -e^{-x} \Rightarrow$$

$$u = -\int e^{-x}dx = -(-e^{-x}) + c = e^{-x} + c \Rightarrow ze^{-2x} = e^{-x} + c \Rightarrow z = \frac{e^{-x}+c}{e^{-2x}} = e^x + ce^{2x} = e^x + ce^{2x}$$

$$z = y^{-1} \Rightarrow y = z^{-1} \Rightarrow y = (e^x + ce^{2x})^{-1} = \frac{1}{e^x + ce^{2x}} \Rightarrow \begin{cases} y = \frac{1}{e^x+ce^{2x}} \\ y\equiv 0 \end{cases}$$

$$(b) 4 = \frac{1}{e^{-2} + ce^{-4}} \Rightarrow 4e^{-2} + c\cdot 4e^{-4} = 1 \Rightarrow c = \frac{1-4e^{-2}}{4e^{-4}} = \frac{e^4}{4} - e^2$$

$$(c)\ y = \frac{1}{e^x + ce^{2x}} \Rightarrow y' = \frac{-e^x - 2ce^{2x}}{(e^x+ce^{2x})^2} = -\frac{e^x(1+2ce^x)}{e^{2x}(1+ce^x)^2} = -\frac{1+2ce^x}{e^x(1+ce^x)^2}$$

$$y' + 2y = y^2e^x \Rightarrow y' = y^2e^x - 2y = \left(\frac{1}{e^x+ce^{2x}}\right)^2 e^x - \frac{2}{e^x+ce^{2x}} = \frac{e^x - 2e^x - 2ce^{2x}}{(e^x+ce^{2x})^2} = \frac{e^x(1-2-2ce^{2x})}{e^{2x}(1+ce^x)^2} = -\frac{1+2ce^{2x}}{e^x(1+ce^x)^2}$$

14. $(x+1)(y' + 2y^2) = -y$:

$$(a)\ y' + 2y^2 = -\frac{1}{x+1}y \Rightarrow y' + \frac{1}{x+1}y = -2y^2 \Rightarrow y^{-2}y' + \underbrace{\frac{1}{x+1}}_{p}y^{-1} = \underbrace{-2}_{q};$$

$$z = y^{-1} \Rightarrow z' = -y^{-2}y' \Rightarrow -z' = y^{-2}y' \Rightarrow -z' + \frac{1}{1+x}z = -2 \Rightarrow z' - \underbrace{\frac{1}{1+x}}_{p}z = \underbrace{2}_{q};$$

$$u = ze^{\int -\frac{1}{1+x}dx} = ze^{-\ln(1+x)} = \frac{1}{1+x}z \Rightarrow u' = -\frac{1}{(1+x)^2}z + \frac{1}{1+x}z' = \frac{1}{1+x}\Big(\underbrace{z' - \frac{1}{1+x}}_{=q=2}\Big) = \frac{2}{1+x} \Rightarrow u = \int\frac{2}{1+x}dx =$$

$$2\ln|1+x| + c \Rightarrow \frac{1}{1+x}z = 2\ln|1+x| + c \Rightarrow z = (1+x)(2\ln|1+x| + c) \Rightarrow y = \frac{1}{(1+x)(2\ln|1+x|+c)},\ or\ y\equiv 0$$

(b) ...
(c) ...

15. $(3x^2y + 2xy + y^3)dx + (x^2 + y^2)dy = 0$ (היתה טעות בשאלה בקובץ השאלות):

$(a)\ (\underbrace{3x^2y + 2xy + y^3}_{M})dx + (\underbrace{x^2 + y^2}_{N})dy = 0 \Rightarrow\ \ M_y = 3x^2 + 2x + 3y^2, N_x = 2x$

$\dfrac{M_y - N_x}{N} = \dfrac{3x^2 + 2x + 3y^2 - 2x}{x^2 + y^2} = 3 = p(x) \Rightarrow \mu = e^{\int 3dx} = e^{3x} \Rightarrow$

$e^{3x}(\underbrace{3x^2y + 2xy + y^3}_{M})dx + e^{3x}(\underbrace{x^2 + y^2}_{N})dy = 0\ is\ exact;\ \ u(x,y) = c\ is\ a\ solution, where:\ \ u = \int M(x,y)dx + \varphi(y) \Rightarrow$

$u = \int e^{3x}(3x^2y + 2xy + y^3)dx + \varphi(y) = y\int (\underbrace{3e^{3x}x^2 + e^{3x}\cdot 2x}_{=(e^{3x}\cdot x^2)'})dx + \int e^{3x}y^3dx + \varphi(y) = e^{3x}x^2y + \dfrac{1}{3}e^{3x}y^3 + \varphi(y)$

$u_y = e^{3x}x^2 + e^{3x}y^2 + \varphi'(y) = N = e^{3x}(x^2 + y^2) \Rightarrow \varphi'(y) = 0 \Rightarrow \varphi(y) = c' \Rightarrow$

$u = e^{3x}x^2y + \dfrac{1}{3}e^{3x}y^3 + c \Rightarrow solution:\ \ e^{3x}\left(x^2y + \dfrac{1}{3}y^3\right) = c$

16. $(e^{2y} + x - 1)y' = 1$:

$(a)\ 1\cdot dx + (1 - e^{2y} - x)dy = 0 \Rightarrow\ \ \ M_y = 0,\ \ N_x = -1 \Rightarrow not\ exact$

$\dfrac{M_y - N_x}{N} = \dfrac{1}{1 - e^{2y} - x} \neq p(x);\ \ \dfrac{N_x - M_y}{M} = -\dfrac{1}{1} = -1 = q(y) \Rightarrow \mu = e^{\int -1dy} = e^{-y} \Rightarrow$

$e^{-y}dx + (e^{-y} - e^{y} - xe^{-y})dy = 0\ is\ exact;\ \ \ u(x,y) = c\ \ is\ a\ solution, where:\ \ u = \int M(x,y)dx + \varphi(y) \Rightarrow$

$u = \int e^{-y}dx + \varphi(y) = xe^{-y} + \varphi(y) \Rightarrow u_y = -xe^{-y} + \varphi'(y) = N = e^{-y} - e^{y} - xe^{-y} \Rightarrow \varphi'(y) = e^{-y} - e^{y} \Rightarrow$

$\varphi(y) = \int (e^{-y} - e^{y})dy = -e^{-y} - e^{y} \Rightarrow u = xe^{-y} - e^{-y} - e^{y} \Rightarrow solution:\ \ e^{-y} + e^{y} - xe^{-y} = c$

17. $dx + \left(\frac{x}{y} - sin\ y\right)dy = 0$:

$M_y = 0, N_x = \dfrac{1}{y} \Rightarrow not\ exact;\ \ \dfrac{M_y - N_x}{N} = -\dfrac{1}{y} \neq p(x);\ \ \dfrac{N_x - M_y}{M} = \dfrac{1}{y} = q(y) \Rightarrow \mu = e^{\int \frac{1}{y}dy} = e^{ln|y|} = y \Rightarrow$

$ydx + (x - ysiny)dy = 0\ \ is\ exact \Rightarrow u = \int ydx + \varphi(y) = xy + \varphi(y);\ \ u_y = x + \varphi'(y) = N = x - ysiny \Rightarrow \varphi' = -ysiny \Rightarrow$

$\varphi(y) = \int -y\cdot sin\,y\,dy = \begin{bmatrix} u = y \Rightarrow u' = 1 \\ v' = -sin\,y \Rightarrow v = cos\,y \end{bmatrix} = ycosy - \int cos\,y\,dy = ycosy - siny \Rightarrow u = xy + ycosy - siny \Rightarrow$

$solution:\ xy + ycosy - siny = c$

18. $ydx + (2xy - e^{-2y})dy = 0$:

$M_y = 1, N_x = 2y \Rightarrow not\ exact;\ \dfrac{M_y - N_x}{N} = \dfrac{1 - 2y}{2xy - e^{-2y}} \neq p(x);\ \dfrac{N_x - M_y}{M} = \dfrac{2y - 1}{y} = 2 - \dfrac{1}{y} = q(y) \Rightarrow \mu = e^{\int 2 - \frac{1}{y}dy} =$

$e^{2y - ln\,y} = e^{2y}\cdot e^{ln\frac{1}{y}} = \dfrac{1}{y}e^{2y} \Rightarrow e^{2y}dx + \left(2xe^{2y} - \dfrac{1}{y}\right)dy = 0\ \ is\ exact \Rightarrow u = \int e^{2y}dx + \varphi(y) = xe^{2y} + \varphi(y);$

$u_y = 2xe^{2y} + \varphi'(y) = N = 2xe^{2y} - \dfrac{1}{y} \Rightarrow \varphi'(y) = -\dfrac{1}{y} \Rightarrow \varphi(y) = \int -\dfrac{1}{y}dy = -ln\,|y| \Rightarrow u = xe^{2y} - ln\,|y| \Rightarrow xe^{2y} - ln\,|y| = c$

19. $\left(3x + \frac{6}{y}\right)dx + \left(\frac{x^2}{y} + \frac{3y}{x}\right)dy = 0$:

$(3x^2y + 6x)dx + (x^3 + 3y^2)dy = 0 \Rightarrow M_y = 3x^2, N_x = 3x^2 \Rightarrow u = \int (3x^2y + 6x)dx + \varphi(y) = x^3y + 3x^2 + \varphi(y);$

$u_y = x^3 + \varphi'(y) = N = x^3 + 3y^2 \Rightarrow \varphi'(y) = 3y^2 \Rightarrow \varphi(y) = \int 3y^2dy = y^3 \Rightarrow u = x^3y + 3x^2 + y^3 \Rightarrow x^3y + 3x^2 + y^3 = c$