

$$: y' = y.1$$

$$(a) \frac{dy}{dx} = 1 \cdot y \Rightarrow \int \left(\frac{1}{y}\right) dy = \int dx \Rightarrow \ln|y| = x + c' \Rightarrow y = e^{x+c'} \xrightarrow{e^c := c} y = c \cdot e^x$$

$$(b) y(-2) = 4 \Rightarrow 4 = c \cdot e^{-2} \Rightarrow c = 4e^2 \Rightarrow y = 4e^{x+2}$$

$$(c) y' = y: y' = (c \cdot e^x)' = c \cdot e^x = y$$

$$: x^2 y' + y = 0.2$$

$$(a) x^2 y' + y = 0 \Rightarrow y' = -\frac{y}{x^2} \Rightarrow \frac{dy}{dx} = -\frac{y}{x^2} \Rightarrow \frac{1}{y} dy = -\frac{1}{x^2} dx \Rightarrow \int \frac{1}{y} dy = -\int \frac{1}{x^2} dx \Rightarrow \ln|y| = -\left(-\frac{1}{x}\right) + c' = \frac{1}{x} + c' \Rightarrow$$

$$y = e^{\frac{1}{x} + c'} \xrightarrow{c := e^c} y = c \cdot e^{\frac{1}{x}}$$

$$(b) y(-2) = 4 \Rightarrow 4 = c \cdot e^{-\frac{1}{2}} \Rightarrow c = 4\sqrt{e} \Rightarrow y = 4 \cdot e^{\frac{1}{x} + \frac{1}{2}}$$

$$(c) y' = -\frac{y}{x^2}: y' = \left(c \cdot e^{\frac{1}{x}}\right)' = c \cdot e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) = -\frac{c \cdot e^{\frac{1}{x}}}{x^2} = -\frac{y}{x^2}$$

$$: (y+1)xy' - (x-1)y = 0.3$$

$$(a) y' = \frac{(x-1)y}{(y+1)x} \Rightarrow \frac{dy}{dx} = \frac{(x-1)y}{(y+1)x} \Rightarrow \int \left(1 + \frac{1}{y}\right) dy = \int \left(1 - \frac{1}{x}\right) dx \Rightarrow y + \ln|y| = x - \ln|x| + c' \Rightarrow \ln|y| + \ln|x| = x - y + c' \Rightarrow \ln|xy| = x - y + c' \Rightarrow xy = e^{x-y+c'} \xrightarrow{c := e^c} xy = c \cdot e^{x-y}$$

$$(b) y(-2) = 4 \Rightarrow -2 \cdot 4 = c \cdot e^{-2-4} \Rightarrow c = -8e^6 \Rightarrow xy = -8e^{x-y+6}$$

$$(c) y' = \frac{(x-1)y}{(y+1)x}; xy = ce^{x-y} \Rightarrow y = \frac{ce^{x-y}}{x} \Rightarrow y' = \frac{x \cdot ce^{x-y} \cdot (x-y)' - ce^{x-y}}{x^2} = \frac{xce^{y-x}(1-y') - ce^{y-x}}{x^2} = \frac{ce^{x-y}(x-xy' - 1)}{x^2} = \frac{y(x-xy' - 1)}{x} \Rightarrow xy' = xy - xyy' - y \Rightarrow y'(x+xy) = xy - y \Rightarrow y' = \frac{y(x-1)}{x(y+1)}$$

$$: (y+x)y' + y - x = 0.4$$

$$(a) (y+x) \cdot \frac{dy}{dx} + (y-x) = 0 \Rightarrow (y-x)dx + (y+x)dy = 0; M_y = 1, N_x = 1 \Rightarrow \text{the equation is exact} \Rightarrow$$

$$u(x,y) = \int M dx + \varphi(y) = \int (y-x) dx + \varphi(y) = xy - \frac{1}{2}x^2 + \varphi(y);$$

$$u_y = x + \varphi'(y) = N = y + x \Rightarrow \varphi'(y) = y \Rightarrow \varphi(y) = \frac{1}{2}y^2 \Rightarrow \text{solution: } u = xy - \frac{1}{2}x^2 + \frac{1}{2}y^2 = c' \Rightarrow 2xy - x^2 + y^2 = c$$

$$(b) 2 \cdot (-2) \cdot 4 - 22 + 4^2 = c \Rightarrow c = -4$$

$$(c) F(x,y) = 2xy - x^2 + y^2; y' = -\frac{F_x}{F_y} = -\frac{2y-2x}{2x+2y} = -\frac{y-x}{x+y} \Rightarrow (y+x)y' = -(y-x) \Rightarrow (y+x)y' + y - x = 0$$

$$: xy' - y = \sqrt{x^2 + y^2}.5$$

$$(a) xy' - y = \sqrt{x^2 + y^2} \Rightarrow y' = \sqrt{1 + \left(\frac{y}{x}\right)^2} + \frac{y}{x}; u = \frac{y}{x} \Rightarrow y = xu \Rightarrow y' = u + xu' \Rightarrow u + xu' = \sqrt{1 + u^2} + u \Rightarrow$$

$$x \cdot \frac{du}{dx} = \sqrt{1 + u^2} \Rightarrow \frac{1}{\sqrt{1 + u^2}} du = \frac{1}{x} dx \Rightarrow \int \frac{1}{\sqrt{1 + u^2}} du = \int \frac{1}{x} dx \Rightarrow \operatorname{arcsinh}(u) = \ln(u + \sqrt{u^2 + 1}) = \ln|x| + \ln c \xrightarrow{c := \ln c} = \ln|xc|$$

$$\Rightarrow u + \sqrt{u^2 + 1} = xc; y = ux \Rightarrow cy = u^2 + u\sqrt{u^2 + 1} = \frac{y^2}{x^2} + \frac{y}{x} \sqrt{\frac{y^2}{x^2} + 1} \Rightarrow x^2 cy = y^2 + xy\sqrt{y^2/x^2 + 1} \Rightarrow$$

$$cx^2 - y = x\sqrt{y^2/x^2 + 1} \Rightarrow c^2 x^4 - 2cx^2 y + y^2 = x^2 \left(\frac{y^2}{x^2} + 1\right) = x^2 + y^2 \Rightarrow c^2 x^4 - 2cx^2 y = x^2 \Rightarrow c^2 x^2 - 2cy = 1 \Rightarrow$$

$$y = \frac{c^2 x^2 - 1}{2c}$$

$$(b) y(-2) = 4 \Rightarrow 4 = \frac{4c^2 - 1}{2c} \Rightarrow 4c^2 - 8c - 1 = 0 \Rightarrow c = 1 \pm \sqrt{5/4}$$

$$(c) y' = \frac{\sqrt{x^2 + y^2} + y}{x}; \left(\frac{c^2 x^2 - 1}{2c}\right)' = \frac{2c^2 x \cdot 2c - 0}{4c^2} = \frac{2c^2 x}{2c} = cx =: (*)$$

$$\frac{\sqrt{x^2+y^2}+y}{x} = \frac{1}{x} \left( \sqrt{x^2 + \left(\frac{c^2x^2-1}{2c}\right)^2} + \frac{c^2x^2-1}{2c} \right) = \frac{1}{2xc} \left( \sqrt{4c^2x^2 + c^4x^4 - 2c^2x^2 + 1 + c^2x^2 - 1} \right) =$$

$$\frac{1}{2xc} \left( \sqrt{(c^2x^2)^2 + 2c^2x^2 + 1 + c^2x^2 - 1} \right) = \frac{1}{2xc} \left( \sqrt{(c^2x^2+1)^2 + c^2x^2 - 1} \right) = \frac{1}{2xc} (c^2x^2 + 1 + c^2x^2 - 1) = cx = (*)$$

$$: y^2 + x^2y' = xyy' \quad .6$$

$$y^2 + x^2y' = xyy' \Rightarrow y' = \frac{y^2}{xy - x^2} \Rightarrow y' = \frac{(y/x)^2}{y/x - 1}; \quad u = \frac{y}{x} \Rightarrow y = xu \Rightarrow y' = u + xu'$$

$$y' = u + xu' = \frac{u^2}{u-1} \Rightarrow x \cdot \frac{du}{dx} = \frac{u^2 - u^2 + u}{u-1} = \frac{u}{u-1} \Rightarrow \left(1 - \frac{1}{u}\right) du = \frac{1}{x} dx \Rightarrow \int 1 - \frac{1}{u} du = \int \frac{1}{x} dx \Rightarrow u - \ln|u| = \ln|x| + c$$

$$\Rightarrow \frac{y}{x} - \ln \frac{y}{x} = \ln x + c \Rightarrow \frac{y}{x} = \ln x + \ln \frac{y}{x} + c = \ln y + c \Rightarrow \ln y = \frac{y}{x} - c \Rightarrow y = c' \cdot e^{\frac{y}{x}}$$

$$(b) y(-2) = 4 \Rightarrow 4 = c' \cdot e^{-2} \Rightarrow c' = 4e^2 \Rightarrow y = 4e^{\frac{y}{x}+2}$$

$$(c) - \quad : (x^2 + y^2)y' = 2xy \quad .7$$

$$(a) (x^2 + y^2)y' = 2xy \Rightarrow y' \left(1 + \left(\frac{y}{x}\right)^2\right) = 2 \cdot \frac{y}{x}; \quad u = \frac{y}{x} \Rightarrow y' (1 + u^2) = 2u \Rightarrow y' = \frac{2u}{1 + u^2} \Rightarrow u + xu' = \frac{2u}{1 + u^2} \Rightarrow$$

$$x \cdot \frac{du}{dx} = \frac{2u - u - u^3}{1 + u^2} = \frac{u - u^3}{1 + u^2} \Rightarrow \frac{1 + u^2}{u - u^3} du = \frac{1}{x} dx \Rightarrow \int \frac{1}{u - u^3} du + \int \frac{u}{1 - u^2} du = \int \frac{1}{x} dx \Rightarrow -\ln(u+1) + \ln u - \ln(u-1)$$

$$= \ln x + c' \Rightarrow \ln \frac{u}{u^2 - 1} = \ln xc'' \Rightarrow xc'' = \frac{u}{u^2 - 1} = \frac{y/x}{(y/x)^2 - 1} \Rightarrow \frac{y^2}{x} c'' - xc'' = \frac{y}{x} \Rightarrow y^2 c'' - x^2 c'' = y \Rightarrow y^2 - x^2 = cy$$

$$(b) 16 - 4 = 4c \Rightarrow c = 3 \Rightarrow y^2 - x^2 = 3y$$

$$(c) y' = \frac{2xy}{x^2 + y^2}; \quad y^2 - x^2 = cy \Rightarrow y = \frac{y^2 - x^2}{c} = \frac{1}{c} y^2 - \frac{1}{c} x^2 \Rightarrow y' = \frac{1}{c} (y^2)' - \frac{1}{c} (x^2)' = \frac{1}{c} \cdot 2yy' - \frac{1}{c} 2x \Rightarrow$$

$$y' \left(1 - \frac{2y}{c}\right) = -\frac{2x}{c} \Rightarrow y' = \frac{-2x}{c - 2y} = \frac{2xy}{2y^2 - cy} = \frac{2xy}{2y^2 - y^2 + x^2} = \frac{2xy}{x^2 + y^2}$$

$$: y' - \frac{2y}{x+1} = (x+1)^3 \quad .8$$

$$(a) y' + \left(-\frac{2}{x+1}\right)y = (x+1)^3 \Rightarrow \text{linear equation in 1st order} \Rightarrow z(x) = ye^{\int p(x)dx} = ye^{\int -\frac{2}{x+1}dx} = ye^{-2 \cdot \ln(x+1)} = ye^{\ln \frac{1}{(x+1)^2}}$$

$$\Rightarrow z(x) = \frac{y}{(x+1)^2} \Rightarrow z'(x) = y' \cdot \frac{1}{(x+1)^2} + y \left(-\frac{2}{(x+1)^3}\right) = \frac{1}{(x+1)^2} \cdot \left(y' - \frac{2y}{x+1}\right) = \frac{(x+1)^3}{(x+1)^2} = x+1 \Rightarrow$$

$$z(x) = \int x+1 dx = \frac{x^2}{2} + x + c'; \quad y \frac{1}{(x+1)^2} = \frac{x^2 + 2x}{2} + c' = \frac{(x+1)^2}{2} - \frac{1}{2} + c' = \frac{(x+1)^2}{2} + c \Rightarrow y = \frac{(x+1)^4}{2} + c(x+1)^2$$

$$(b) 4 = \frac{1}{2} + c \Rightarrow c = 3.5 \Rightarrow y = \frac{(x+1)^4}{2} + 3.5(x+1)^2$$

$$(c) y' = (x+1)^3 + \frac{2y}{x+1} = (x+1)^3 + \frac{2}{x+1} \left(\frac{(x+1)^4}{2} + c(x+1)^2\right) = (x+1)^3 + (x+1)^3 + 2c(x+1) =$$

$$2(x+1)^3 + 2c(x+1); \quad y = \frac{(x+1)^4}{2} + c(x+1)^2 \Rightarrow y' = 2(x+1)^3 + 2c(x+1)$$

$$: y + y' = e^{-x} \quad .9$$

$$(a) y' + 1 \cdot y = e^{-x} \Rightarrow z(x) = ye^{\int dx} = ye^x \Rightarrow z'(x) = y' e^x + ye^x = e^x (y' + y) = 1 \Rightarrow z(x) = x + c \Rightarrow$$

$$ye^x = x + c \Rightarrow y = \frac{x+c}{e^x}$$

$$(b) 4 = (-2+c)e^2 \Rightarrow c = \frac{4}{e^2} + 2 \dots$$

$$(c) y = \frac{x+c}{e^x} \Rightarrow y' = \frac{e^x - (x+c)e^x}{e^{2x}} = \frac{1-x-c}{e^x}; \quad y + y' = e^{-x} \Rightarrow y' = \frac{1}{e^x} - y = \frac{1}{e^x} - \frac{x+c}{e^x} = \frac{1-x-c}{e^x}$$

$$: (1+x)y' - [2y + (x+1)^4] = 0 \quad .10$$

$$(1+x)y' - 2y - (x+1)^4 = 0 \Rightarrow y' - \frac{2}{x+1}y = (x+1)^3 \Rightarrow \text{פתרון תרגיל 8}$$

$$: x \ln x \cdot y' - y = x^3(3 \ln x - 1) \quad .11$$

$$y' - \frac{1}{(x \ln x)} y = \frac{3x^3 \ln x - x^3}{x \ln x} = 3x^2 - \frac{x^2}{\ln x}; \quad z(x) = ye^{\int p(x)dx} = ye^{\int -\frac{1}{x \ln x} dx}$$

$$\int -\frac{1}{x \ln x} dx = -\int \frac{1}{\ln x} \cdot \frac{1}{x} dx = -\int \frac{1}{\ln x} \cdot (\ln x)' dx = \left[ \begin{array}{l} f \equiv \ln \\ g \equiv \ln \end{array} \right] = -\int f'(g(x)) \cdot g'(x) dx = -\ln \ln x \Rightarrow$$

$$z(x) = ye^{-\ln \ln x} = \frac{1}{\ln x} y \Rightarrow z'(x) = \frac{1}{\ln x} y' - \frac{1}{x(\ln x)^2} y = \frac{1}{\ln x} \left( y' - \frac{1}{x \ln x} y \right) = \frac{1}{\ln x} \left( 3x^2 - \frac{x^2}{\ln x} \right) = \frac{3x^2}{\ln x} - \frac{x^2}{(\ln x)^2} \Rightarrow$$

$$z(x) = \int \frac{3x^2}{\ln x} - \frac{x^2}{(\ln x)^2} dx = \int \frac{3x^2 \ln x - x^2}{(\ln x)^2} dx = \int \frac{3x^2 \cdot \ln x - x^3 \cdot \frac{1}{x}}{(\ln x)^2} dx = \left[ \begin{array}{l} f \equiv x^3 \\ g \equiv \ln x \end{array} \right] = \int \left( \frac{f}{g} \right)' dx = \frac{f}{g} + c = \frac{x^3}{\ln x} + c \Rightarrow$$

$$\frac{x^3}{\ln x} + c = \frac{1}{\ln x} y \Rightarrow y = x^3 + c \ln x$$

(b) ...

(c)

$$: s' \cos t + s \cdot \sin t = 1.12$$

$$(a) s' + s \cdot \frac{\sin t}{\cos t} = \frac{1}{\cos t}; \quad z(t) = s(t) \cdot e^{\int \frac{\sin t}{\cos t} dt}$$

$$\int \frac{\sin t}{\cos t} dt = -\int \frac{1}{\cos t} \cdot (-\sin t) dt = \left[ \begin{array}{l} f'(t) \equiv \frac{1}{t} \Rightarrow f \equiv \ln t \\ g'(t) = -\sin t \Rightarrow g(t) = \cos t \end{array} \right] = -\int f'(g(t)) \cdot g'(t) dt = -f \cdot g(t) + c = -\ln(\cos t) + c$$

$$\Rightarrow z = se^{-\ln \cos t} = se^{\ln \frac{1}{\cos t}} = \frac{1}{\cos t} s \Rightarrow z' = \frac{\sin t}{(\cos t)^2} s + \frac{1}{\cos t} s' = \frac{1}{\cos t} \left( s' + \frac{\sin t}{\cos t} s \right) = \frac{1}{(\cos t)^2} \Rightarrow$$

$$z = \int \frac{1}{(\cos t)^2} dt = \tan t + c \Rightarrow \frac{1}{\cos t} s = \frac{\sin t}{\cos t} + c \Rightarrow s = \sin t + c \cdot \cos t$$

(b) ...

(c) ...

$$: y' + 2y = y^2 e^x. 13$$

$$(a) y^{-2} y' + 2y^{-1} = e^x; \quad z = y^{-1} \Rightarrow z' = -y^{-2} y' \Rightarrow -z' = y^{-2} y'$$

$$-z' + 2z = e^x \Rightarrow z' + (-2)z = -e^x; \quad u = ze^{\int -2 dx} = ze^{-2x} \Rightarrow u' = z' e^{-2x} - 2ze^{-2x} = e^{-2x} (z' - 2z) = e^{-2x} (-e^x) = -e^{-x} \Rightarrow$$

$$u = -\int e^{-x} dx = -(-e^{-x}) + c = e^{-x} + c \Rightarrow ze^{-2x} = e^{-x} + c \Rightarrow z = \frac{e^{-x} + c}{e^{-2x}} = e^x + ce^{2x} = e^x + ce^{2x}$$

$$z = y^{-1} \Rightarrow y = z^{-1} \Rightarrow y = (e^x + ce^{2x})^{-1} = \frac{1}{e^x + ce^{2x}} \Rightarrow \begin{cases} y = \frac{1}{e^x + ce^{2x}} \\ y \equiv 0 \end{cases}$$

$$(b) 4 = \frac{1}{e^{-2} + ce^{-4}} \Rightarrow 4e^{-2} + c \cdot 4e^{-4} = 1 \Rightarrow c = \frac{1 - 4e^{-2}}{4e^{-4}} = \frac{e^4}{4} - e^2$$

$$(c) y = \frac{1}{e^x + ce^{2x}} \Rightarrow y' = \frac{-e^x - 2ce^{2x}}{(e^x + ce^{2x})^2} = -\frac{e^x(1 + 2ce^x)}{e^{2x}(1 + ce^x)^2} = -\frac{1 + 2ce^x}{e^x(1 + ce^x)^2}$$

$$y' + 2y = y^2 e^x \Rightarrow y' = y^2 e^x - 2y = \left( \frac{1}{e^x + ce^{2x}} \right)^2 e^x - \frac{2}{e^x + ce^{2x}} = \frac{e^x - 2e^x - 2ce^{2x}}{(e^x + ce^{2x})^2} = \frac{e^x(1 - 2 - 2ce^{2x})}{e^{2x}(1 + ce^x)^2} = -\frac{1 + 2ce^{2x}}{e^x(1 + ce^x)^2}$$

$$: (x+1)(y' + 2y^2) = -y. 14$$

$$(a) y' + 2y^2 = -\frac{1}{x+1} y \Rightarrow y' + \frac{1}{x+1} y = -2y^2 \Rightarrow y^{-2} y' + \frac{1}{x+1} y^{-1} = -2;$$

$$z = y^{-1} \Rightarrow z' = -y^{-2} y' \Rightarrow -z' = y^{-2} y' \Rightarrow -z' + \frac{1}{1+x} z = -2 \Rightarrow z' - \frac{1}{1+x} z = 2;$$

$$u = ze^{\int -\frac{1}{1+x} dx} = ze^{-\ln(1+x)} = \frac{1}{1+x} z \Rightarrow u' = -\frac{1}{(1+x)^2} z + \frac{1}{1+x} z' = \frac{1}{1+x} \left( z' - \frac{1}{1+x} z \right) = \frac{2}{1+x} \Rightarrow u = \int \frac{2}{1+x} dx =$$

$$2 \ln |1+x| + c \Rightarrow \frac{1}{1+x} z = 2 \ln |1+x| + c \Rightarrow z = (1+x)(2 \ln |1+x| + c) \Rightarrow y = \frac{1}{(1+x)(2 \ln |1+x| + c)}, \text{ or } y \equiv 0$$

(b) ...  
(c) ...

(a)  $(3x^2y + 2xy + y^3)dx + (x^2 + y^2)dy = 0$   $\Rightarrow M_y = 3x^2 + 2x + 3y^2, N_x = 2x$  : היתה טעות בשאלה בקובץ השאלות) 0.15

$$\frac{M_y - N_x}{N} = \frac{3x^2 + 2x + 3y^2 - 2x}{x^2 + y^2} = 3 = p(x) \Rightarrow \mu = e^{\int 3dx} = e^{3x} \Rightarrow$$

$$e^{3x}(3x^2y + 2xy + y^3)dx + e^{3x}(x^2 + y^2)dy = 0 \text{ is exact; } u(x, y) = c \text{ is a solution, where: } u = \int M(x, y)dx + \varphi(y) \Rightarrow$$

$$u = \int e^{3x}(3x^2y + 2xy + y^3)dx + \varphi(y) = y \int (3e^{3x}x^2 + e^{3x} \cdot 2x)dx + \int e^{3x}y^3dx + \varphi(y) = e^{3x}x^2y + \frac{1}{3}e^{3x}y^3 + \varphi(y)$$

$$u_y = e^{3x}x^2 + e^{3x}y^2 + \varphi'(y) = N = e^{3x}(x^2 + y^2) \Rightarrow \varphi'(y) = 0 \Rightarrow \varphi(y) = c' \Rightarrow$$

$$u = e^{3x}x^2y + \frac{1}{3}e^{3x}y^3 + c \Rightarrow \text{solution: } e^{3x}\left(x^2y + \frac{1}{3}y^3\right) = c$$

$$: (e^{2y} + x - 1)y' = 1.16$$

(a)  $1 \cdot dx + (1 - e^{2y} - x)dy = 0 \Rightarrow M_y = 0, N_x = -1 \Rightarrow \text{not exact}$

$$\frac{M_y - N_x}{N} = \frac{1}{1 - e^{2y} - x} \neq p(x); \frac{N_x - M_y}{M} = -\frac{1}{1} = -1 = q(y) \Rightarrow \mu = e^{\int -1dy} = e^{-y} \Rightarrow$$

$$e^{-y}dx + (e^{-y} - e^y - xe^{-y})dy = 0 \text{ is exact; } u(x, y) = c \text{ is a solution, where: } u = \int M(x, y)dx + \varphi(y) \Rightarrow$$

$$u = \int e^{-y}dx + \varphi(y) = xe^{-y} + \varphi(y) \Rightarrow u_y = -xe^{-y} + \varphi'(y) = N = e^{-y} - e^y - xe^{-y} \Rightarrow \varphi'(y) = e^{-y} - e^y \Rightarrow$$

$$\varphi(y) = \int (e^{-y} - e^y)dy = -e^{-y} - e^y \Rightarrow u = xe^{-y} - e^{-y} - e^y \Rightarrow \text{solution: } e^{-y} + e^y - xe^{-y} = c$$

$$: dx + \left(\frac{x}{y} - \sin y\right)dy = 0.17$$

$$M_y = 0, N_x = \frac{1}{y} \Rightarrow \text{not exact; } \frac{M_y - N_x}{N} = -\frac{1}{y} \neq p(x); \frac{N_x - M_y}{M} = \frac{1}{y} = q(y) \Rightarrow \mu = e^{\int \frac{1}{y}dy} = e^{\ln|y|} = y \Rightarrow$$

$$ydx + (x - y \sin y)dy = 0 \text{ is exact} \Rightarrow u = \int ydx + \varphi(y) = xy + \varphi(y); u_y = x + \varphi'(y) = N = x - y \sin y \Rightarrow \varphi' = -y \sin y \Rightarrow$$

$$\varphi(y) = \int -y \cdot \sin y dy = \left[ \begin{array}{l} u = y \Rightarrow u' = 1 \\ v' = -\sin y \Rightarrow v = \cos y \end{array} \right] = y \cos y - \int \cos y dy = y \cos y - \sin y \Rightarrow u = xy + y \cos y - \sin y \Rightarrow$$

$$\text{solution: } xy + y \cos y - \sin y = c$$

$$: ydx + (2xy - e^{-2y})dy = 0.18$$

$$M_y = 1, N_x = 2y \Rightarrow \text{not exact; } \frac{M_y - N_x}{N} = \frac{1 - 2y}{2xy - e^{-2y}} \neq p(x); \frac{N_x - M_y}{M} = \frac{2y - 1}{y} = 2 - \frac{1}{y} = q(y) \Rightarrow \mu = e^{\int 2 - \frac{1}{y}dy} =$$

$$e^{2y - \ln y} = e^{2y} \cdot e^{-\ln y} = \frac{1}{y}e^{2y} \Rightarrow e^{2y}dx + \left(2xe^{2y} - \frac{1}{y}\right)dy = 0 \text{ is exact} \Rightarrow u = \int e^{2y}dx + \varphi(y) = xe^{2y} + \varphi(y);$$

$$u_y = 2xe^{2y} + \varphi'(y) = N = 2xe^{2y} - \frac{1}{y} \Rightarrow \varphi'(y) = -\frac{1}{y} \Rightarrow \varphi(y) = \int -\frac{1}{y}dy = -\ln|y| \Rightarrow u = xe^{2y} - \ln|y| \Rightarrow xe^{2y} - \ln|y| = c$$

$$: \left(3x + \frac{6}{y}\right)dx + \left(\frac{x^2}{y} + \frac{3y}{x}\right)dy = 0.19$$

$$(3x^2y + 6x)dx + (x^3 + 3y^2)dy = 0 \Rightarrow M_y = 3x^2, N_x = 3x^2 \Rightarrow u = \int (3x^2y + 6x)dx + \varphi(y) = x^3y + 3x^2 + \varphi(y);$$

$$u_y = x^3 + \varphi'(y) = N = x^3 + 3y^2 \Rightarrow \varphi'(y) = 3y^2 \Rightarrow \varphi(y) = \int 3y^2dy = y^3 \Rightarrow u = x^3y + 3x^2 + y^3 \Rightarrow x^3y + 3x^2 + y^3 = c$$