Numerical Analysis 1

Assignment 4

1. Consider a function $f(x) = \sin(\frac{\pi}{2}x)$ defined on the nodes $x_j = \{-3, -2, -1, 0, 1, 2, 3\}$.
   (a) Find Newton's interpolation polynomial $P_5(x)$ for $f(x)$ based on these nodes.
   (b) Find the Taylor's polynomial $T_5(x)$ of $f(x)$ around $x_0 = 0$.
   (c) Use each of the polynomials in (a) and (b) to approximate $f(\frac{5}{3}) = \sin(\frac{5\pi}{6}) = \frac{1}{2}$.
   (d) Which of the two approximations has a smaller error? Explain.

2. Suppose we approximate $f(x) = \sin(3x)$ on $x \in [0, 5]$ using piecewise-linear interpolation on equally-spaced points. How many points are needed to bound the error by 10^{-6}?

3. Suppose we approximate a smooth function $f(x)$ using *piecewise-parabolic* interpolation on equally-spaced points in $[a, b]$, i.e., given a table of $f(x_i)$ we use three table values $\{x_{i-1}, x_i, x_{i+1}\}$ to approximate $f(x)$ inside $[a, b]$ using a second order polynomial $P_2(x)$.
   (a) Find an error bound for $\max_{a \le x \le b} |f(x) - P_2(x)|$.
   (b) Answer question 2 assuming we use piecewise-parabolic interpolation instead of piecewise-linear.

4. Consider the Runge function $f(x) = \frac{1}{1+x^2} \qquad x \in [-5, 5]$. In this question, you will use Lagrange's interpolation to approximate it.
   (a) For a uniform grid of $N + 1$ points, i.e., $x_k = -5 + kh, h = \frac{10}{N}$, use the **polyfit** MATLAB function to find the interpolation polynomial for $N = 5, 11, 21$.
   (b) Using the **polyval** and **plot** MATLAB functions, plot the polynomials of section (a), for values of $x = -5 : 0.01 : 5$.
   (c) Compute the maximal interpolation error.
   (d) Repeat (a-c), using piecewise-linear interpolation, which is given by the MATLAB function **interp1** with the third parameter given as *linear*
   (e) repeat (d), using piecewise cubic interpolation, which is given by the MATLAB function **interp1** with the third parameter given as *pchip*

5. We seek an interpolating polynomial of the function $h(x) = 1/(1+25x^2)$ on the interval $x \in [-5, 5]$ using 11 interpolation points.
   (a) Find (or write a program that finds) the Lagrange polynomial using equally spaced x-values as interpolating points..
   (b) Repeat (a) using the Chebyshev interpolating points.
   (c) Plot $h(x)$ versus the polynomials in (a) and (b).
   (d) Which choice of interpolating points works better?