Computational models – Home Assignment 6

Fall 2008 - 19/1/08 (due: 2/2/09)

1. Write a Context-Free Grammar for each of the following languages:
   a. $L_1=\{ a^i b^j c^k \mid i,j,k\geq 0,\ i+k=j\}$
   b. $L_2=\{ a^i b^j \mid i,j\geq 0,\ i\neq j\}$
   c. $L_3=\{ a^i b^j c^k \mid i,j,k\geq 0,\ i\neq j \text{ or } j\neq k \}$

2. For each of the following languages determine and prove if it is context free:
   a. $L_1=\{ a^n b^{2n} c^n \mid n\in N \}$
   b. $L_2=\{ a^i b^j c^k \mid i,j,k\geq 0,\ k=min(i,j)\}$
   c. $L_3=\{ a^m b^n c^n d^m \mid m,n\in N\}$

3. For each of the following operations determine and prove if the regular languages are closed under the operation and if the context free languages are closed under the operation:
   a. $Reverse(L)= \{w \mid w^R\in L\}$
   b. $PP(L)=\{w \mid w\in L$ and no proper prefix of w is in $L\}$ (i.e., if $w=w_1\ldots w_n$ then a proper prefix of w is a string $w_1\ldots w_i$ where $0<i<n$)
   c. $Half(L)=\{ x \mid \exists y\in\Sigma^*$ s.t. $|x|=|y|$ and $xy\in L \}$

4. Let G be a context free grammar in Chomsky Normal Form (CNF). Prove (formally) that for any $w\in L(G)$, $|w|=n$ ($n>0$), w is derived in exactly $2n-1$ derivations steps.

5. We would like to build algorithms that check if the language of a DFA A or a CFG G is infinite.
   a. Prove that $L(A)$ is infinite iff there exists a word $w\in L(A)$ s.t. $|Q|<|w|\leq 2|Q|$ (Q is the set of states of A).
   b. Conclude that there exists an algorithm deciding whether $L(A)$ is infinite.
   c. Prove that given G, a CFG in CNF, $L(G)$ is infinite iff there exists a word $w\in L(G)$ s.t. $2^{|V|}<|w|\leq 2^{|V|+1}$ (V is the set non-terminals of G)
   d. Conclude that there exists an algorithm deciding whether $L(G)$ is infinite.
   e. Show an algorithm deciding if $|L(A)|=2009$.
   f. Show an algorithm deciding if $|L(G)|=2009$.

6.
   a. Show that the pumping lemma cannot be used to show that the following language is not context-free (i.e., any word in the language can be pumped according to the definitions of the lemma): $L=\{3^n 0^i 1^j 2^k \mid$ if $n=1$ then $i=j=k\}$
   b. Prove that L is not context free.

7. **Bonus (10 points)**: For the languages L_1 and L_2 over an alphabet Σ we define: $A(L_1,L_2)=\{x\in\Sigma^* \mid \exists y,z\in L_2 \text{ s.t. } yxz\in L_1\}$. Assume L_1 is regular and L_2 is context free. Is $A(L_1,L_2)$ regular? Prove.