Computational Models - Exercise 5
19/1/08

1. Write a DFA for each of the following languages above the alphabet $\Sigma=\{a,b\}$:
    a. Σ^*
    b. ϕ
    c. $\{\varepsilon\}$
    d. $\{w \mid |w| \bmod 3 = 0\}$
    e. *{w | w does not contain the substring 'abba'}*

2. For each of the following languages write a NFA and convert it to a DFA:
    a. {*w* | *w* contains the substring 'aa' or doesn't contain the substring 'bab'}
    b. $\{w\in\{a,b\}^*$ | There exists a partitioning of *w* to *w=xy* s.t. *x* contains an even number of 'a's and *y* contains an even number of 'b's}

3. Write a regular expression for each of the following languages above $\Sigma=\{0,1\}$:
    a. $\{w \mid |w| \bmod 4 = 0\}$
    b. {*w* | *w* contains exactly four '1's}

4. For each of the following languages prove that if *L* is regular then *L'* is regular
    a. $L'=\{ xy \mid x\notin L$ and $y\in L\}$
    b. $L'=\{x_2x_4\ldots x_{2n} \mid x_2,x_4,\ldots,x_{2n}\in\Sigma, \exists x_1,x_3,x_{2n-1}\in\Sigma$ s.t. $x_1x_2\ldots x_{2n-1}x_{2n}\in L\}$, i.e. *L'* is the language of all strings that are a concatenation of the characters appearing in even indices in words of even length from *L*.

5. Prove if the following languages are regular:
    a. $L_1=\{ w \mid w\in\{0,1\}^*, w=w^R\}$
    b. $\Sigma=\{0,1,\#\}$, $L_2=\{x\#y\#z \mid x,y,z\in\{0,1\}^*, x+y=z\}$ (where *x,y,z* are numbers in binary representation).
    c. $\Sigma=\{0,1\}$, $L_3=\{x_1y_1z_1\ldots x_ny_nz_n \mid x=x_n\ldots x_1, y=y_n\ldots y_1, z=z_n\ldots z_1, x+y=z\}$, where *x,y,z* are numbers in binary representation and $x_1,\ldots x_n, y_1,\ldots y_n, z_1,\ldots,z_n\in\Sigma$
    d. $\Sigma=\{0,1\}$, $L_4=\{w$ | the number of '01's in *w* is equal to the number of '10's}

6. Space complexity:
    a. Prove that *Clique* is in *PSPACE* by giving an algorithm that solves *Clique* with polynomial space. (*Clique={<G,k>|* G is an undirected graph with a clique of size at least *k}*)
    b. Prove that *QSAT* (shown in class and recitation) is *NP-hard*. (*QSAT={<ϕ>| ϕ is a true quantified Boolean formula}*)