Computational Models - Exercise 4
22/12/08

Remark: Some of the questions might be easier to solve after class no. 9.

1. Prove or disprove: *NP=coNP* iff there exists a problem in *NP-complete* (*NPC*) that is also in *coNP*.

2.
   a. prove or disprove: $Halt \in NP\text{-}hard$
   b. Is it also *NPC*?

3. Determine if the following problems are in *P* or *NPC* and prove correctness:
   a. Input: 3-CNF formula φ
      Question: Is there an assignment satisfying φ where 2008 variables are assigned *true* and the rest of the variables are assigned *false*?
   b. Input: 3-CNF formula φ with an even number of variables
      Question: Is there an assignment satisfying φ where half the variables are assigned *true* and half are assigned *false*?
   c. Input: directed graph $G=(V,E)$
      Question: Is there a Hamiltonian cycle in *G*?
   d. Input: directed graph $G=(V,E)$
      Question: Are there two simple cycles in *G* with disjoint vertices that cover all of the vertices of *G* (a simple cycle is a cycle that doesn't visit the same vertex twice)?
   e. Input: undirected graph $G=(V,E)$ where $|V|$ is even
      Question: Is there a clique in *G* of size at least $|V|/2$?
   f. Input: A set *S*, $|S|=n$, subsets $A_1,A_2,...,A_m \subseteq S$, a natural number *k*
      Question: is there a subset $C \subseteq S$ of size *k* (or smaller), s.t for every $1 \le i \le m$, $C \cap Ai \neq \phi$ (the empty set)?
   g. Input: 3-CNF formula φ
      Question: Is there an assignment satisfying at least half of the clauses in φ?

4. Definition: An undirected graph *G* is **k-colorable** if there is a function $C:V \rightarrow \{1,2,...,k\}$ s.t. for every $(u,v) \in E$, $C(u) \neq C(v)$
   Explanation: A graph is k-colorable if one can color its vertices with *k* colors and every two vertices connected with an edge have different colors.
   We define the following language: $k\text{-}col=\{G=(V,E) \mid G \text{ is k-colorable}\}$.
   It is known that $3\text{-}col \in NPC$. Prove that for any $k \ge 3$, $k\text{-}col \in NPC$

5. We define the class *coNPC*: $L \in coNPC$ if
   a. $L \in coNP$
   b. $\forall A \in coNP, A \le L$
   Prove that if $L \in NPC$ then $L^c \in coNPC$

6. (question from exam) if L∈NP-hard and L∈coNP-hard then necessarily:
    a. $NP=coNP$
    b. $L \notin R$
    c. $L \in RE$
    d. None of the above.

    Prove your answer.