Computational Models - Exercise 2
24/11/08

Guidelines: You may use any claim proved during class or recitation.

1. We would like to prove a slightly more general version of Rice's theorem for the specific case of predicate programs (programs that output *true, false* or diverge). We redefine the notion of a semantic problem in the following way: A semantic decision problem is a problem *P* with a predicate program as input for which the answer only depends on *L(P)*: *if L(p1)=L(p2) then P(p1)=P(p2)*. Prove this slightly more general version of Rice's theorem.

2. In this question we explore the properties of mapping reduction:
    a. We define the following languages: *SORT* = {<*l*> | *l* is a list of sorted numbers}, *HALT--* = {<*p,x*> | *p* is a program in c-- that halts on *x*}. Give a mapping reduction HALT--$\leq_m$SORT. What does this imply on the computability of HALT--?
    b. Why does this reduction fail for HALT?
    c. Let A be a recursive language. Show A$\leq_m$HALT.
    d. Prove or disprove. If there is a reduction from language A to B then there is also a mapping reduction A$\leq_m$B.

3. Prove or disprove. The function $s:N \rightarrow N$ is computable, where:

$s(n) = max \{f_i(j) : i,j \leq n\} + 1$, where $f_0, f_1, ...$ is an ordering over all computable numeric functions.

Hint: recall the diagonalization proofs shown in class.

4. Prove that the class *R* is closed under the following operations (you can use words and not code):
    a. Union (i.e. if $L_1 \in R$ and $L_2 \in R$ then $L_1 \cup L_2 \in R$)
    b. Intersection
    c. Concatenation
    d. Kleene star ($L^* = \cup_{i=0...\propto} L^i$)

5. For the following languages determine whether they belong to R, RE/R, co-RE/R, or none of the above, and prove correctness:
    a. Input: program *p*
       Question: Is there an *x* for which *p* halt?
    b. Input: program *p*
       Question: Is every even number a sum of two prime numbers (Goldbach conjecture)?
    c. Input: a program *p* and two inputs *x* and *y*.
       Question: Does *p* halt on exactly one of the two inputs?
    d. Input: predicate program *p*
       Question: is $|L(p)|>3$?
    e. Input: predicate program *p*
       Question: is $|L(p)| \leq 3$?
    f. Input: a predicate program *p*
       Question: is *L(p)* decidable?
    g. Input: a predicate program *p*
       Question: is *L(p)* semi-decidable?
    h. Input: a program *p* with no input, s.t. $|p|<bb(1000)$
       Question: Does *p* halt?

6. $L_1,L_2 \in RE/R$. Prove whether the following is possible:
    a. $L_1 \cup L_2 \in R$
    b. $L_1 \cup L_2 \in R$ and $L_1 \cap L_2 \in R$