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Proof of Rice's Theorem using the recursion theorem:

Let $P = \{M \mid M\ is\ a\ TM\}$ be a non-trivial property (i.e. $P \neq \Sigma^*, \emptyset$ and $\forall M_1, M_2: L(M_1) = L(M_2) \Rightarrow M_1 \in P \Leftrightarrow M_2 \in P$).

Following is a proof that P is undecidable:

Assume by contradiction that P is decidable, then there exists a TM M that decides it. Let A, B be two TMs such that $A \in P$ and $B \notin P$. Define the following TM R:

For input w:

- Obtain $\langle R \rangle$ (by the recursion theorem).
- Run M on $\langle R \rangle$.
- If M accepts, simulate B on w, otherwise simulate A on w.

For any TM N, either $N \in P$ or $N \notin P$. If $R \in P$, then M accepts $\langle R \rangle$, but then it will simulate B, which is not in P, therefore $L(R) \equiv L(B)$ and R should be $R \notin P$ – contradiction. In a similar fashion we get a contradiction if we start with $R \notin P$. Therefore there exists no M that decides P.