CS525 Winter 2012 \ Class Assignment #1, 1/18/2012

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1.46)

a.

Let $L = \{0^n 1^m 0^n \mid m, n \geq 0\}$. Following is a proof that L is not a regular language:

Assume by contradiction that L is regular, then according to the pumping lemma, there exists $p \in \mathbb{N}$ such that for any $s = xyz \in L$ where $|y| > 0$ and $|xy| \leq p$ the string $xy^i z \in L$ for any $i \geq 0$.

Let $s = 0^p 1 0^p$. Clearly $s \in L$. Because of the given conditions, y must be constructed only of 0's, since $|xy| \leq p$ and the first p characters are 0. Therefore y is of the form $y = 0^k$ where $k \in \{1,2, \dots, p\}$. Now let the pumped string be $xy^2 z$, then for any k as defined, we will get a different number of 0's at the beginning compared to the end ($p - k + 2k$ 0's at the beginning and p 0's at the end). Therefore for the pumped string is not in the language L, thus L is not regular.

c.

Let $L = \{w \mid w \in \{0,1\}^* \textit{ is not a palindrome}\}$. Since the set of regular languages is closed under complement, to show that L is not a regular language, it is sufficient to show that $\bar{L}$ is not a regular language.

$\bar{L} = \{w \mid w \in \{0,1\}^* \textit{ is a palindrome}\}$. We will show that $\bar{L}$ is not regular using the pumping lemma:

Assume by contradiction that $\bar{L}$ is regular, then it satisfies the conditions of the pumping lemma. Let $s = 0^p 1 0^p$ as before, which happens to be a palindrome, thus $s \in \bar{L}$. But we showed in (a) that when pumping $s = xyz$, the pumped string $xy^2 z$ is not in the language (i.e. $0^{p-k+2k} 1 0^p$ is not a palindrome for any $k \in \{1,2, \dots, p\}$). Therefore $\bar{L}$ is not a regular language, thus L is not a regular language.

d.

Let $L = \{wtw \mid w, t \in \{0,1\}^+\}$. Following is a proof that L is not a regular language.

Assume by contradiction that L is regular, then it satisfies the conditions of the pumping lemma. We will choose $s = 0^p 1^p 1 0^p 1^p$. As before, y can only be a sequence of $1, \dots, p$ 0's at the beginning. Looking at the pumped string $xy^2 z$ for any selection of y as a sequence of k 0's as described above, we will receive $0^{p-k+2k} 1^p 1 0^p 1^p$, which is not in L since there exists no partition of the pumped string such that the prefix can be matched to the suffix:

- If w is constructed of 0's and 1's, then for the prefix string we would have more 0's than available for the suffix string (and for the prefix we're taking all 0's so we can take also 1's).
- If w is constructed only of 0's, that cannot be correct, as the suffix ends with p 1's.

Therefore L is not a regular language.

1.48) Wrong answer!

Let $D = \{w \mid w\ contains\ an\ equal\ number\ of\ occurences\ of\ the\ substrings\ 01\ and\ 10\}$

D is regular since it is described in the following regular expression: $(10)^{+}1 \cup (01)^{+}0 \cup \epsilon$.

1.54)

a.

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b.

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c.

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