Homework 6
Due 02/06/2009

1. Let 10TSP be a variant of TSP, in which instead of triangle inequality, for every three vertices $x, y, z \in V$ we have $w(x, y) \leq w(x, z) + 10 \cdot w(z, y)$. Suppose we apply the 2-approximation algorithm for TSP-with-triangle-inequality from class on 10TSP instances. What is the approximation ratio we get? Try to make your analysis tight.

2. Consider the greedy algorithm for the Vertex Cover problem: Choose a vertex of maximal degree. Add it to the cover, and remove it and all its edges from the graph. Repeat until no edges remain in the graph. Prove that this algorithm is *not* a 2-approximation.

3. (a) Prove that graphs with maximum degree Δ are $(\Delta + 1)$-colorable, and show a polynomial-time algorithm that finds a $(\Delta + 1)$-coloring.
   (b) Prove that if there exists a 2-approximation polynomial-time algorithm for the maximum independent set problem, then there exists a polynomial-time algorithm that colors a given 3-colorable graph using $O(\log n)$ colors, where n denotes the number of the vertices.
   (c) Show a polynomial-time algorithm that finds an $O(\sqrt{n})$-coloring of a 3-colorable graph, where n denotes the number of the vertices. Hint: Use (3a) and the fact that we can find a 2-coloring of a 2-colorable graph in polynomial-time.

4. Provide a new 2-approximation for the *weighted* VC problem as follows. State the problem as a linear programming problem, and show that rounding the obtained solution gives the desired approximation ratio.

5. Prove that there is a constant $c > 1$ such that it is impossible to approximate MAX-5-SAT in polynomial-time within the factor c unless $\mathsf{P} = \mathsf{NP}$.